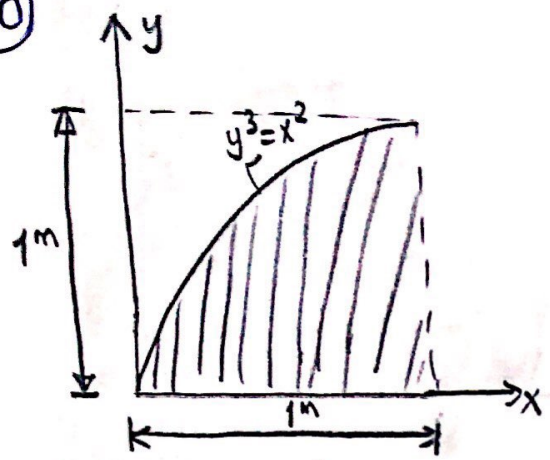
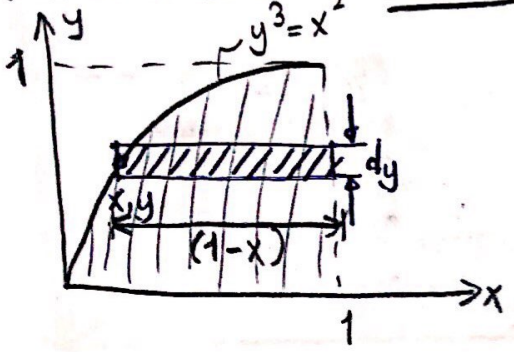
(10)




Şekilde verilen taralı alanın x ve y eksenleri doğrultusundaki atalet momentlerini hesaplayınız.

* dA elemanı seçilir → düşey veya yatay şerit
- Atalet momentinin hesaplanacağı eksene paralel eleman seçmek,
- y, x cinsinden kolayca ifade edilebiliyorsa düşey şerit seçmek

a) Yatay şerit seçmek $\Rightarrow I_x$ için




$$I_x = \int y^2 . dA \quad ; \quad dA = (1-x).dy$$

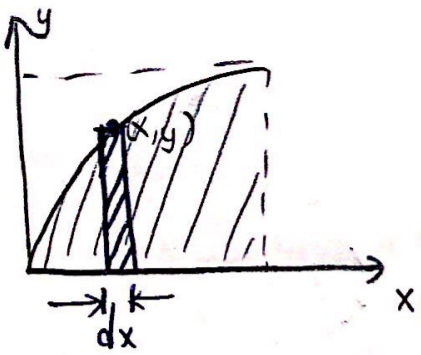
$$x = y^{3/2}$$

$$dA = (1-y^{3/2}).dx$$

$$I_x = \int_0^1 y^2.(1-y^{3/2}).dy = \int_0^1 (y^2 - y^{7/2}).dy$$

$$I_x = \left(\frac{y^3}{3} - \frac{2}{9} y^{9/2}\right)\Big|_0^1 \Rightarrow I_x = 0,111\, m^4 \;//$$

b) Düşey şerit seçmek $\Rightarrow I_y$ için

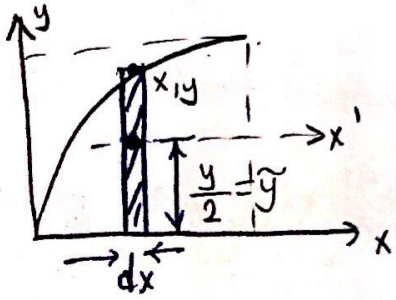



$$I_y = \int x^2 . dA \quad ; \quad dA = y.dx = x^{2/3}.dx$$

$$I_y = \int_0^1 x^2 . x^{2/3} . dx = \int_0^1 x^{8/3} . dx$$

$$I_y = \left(\frac{3}{11} . x^{11/3}\right)\Big|_0^1 \Rightarrow I_y = 0,273\, m^4$$

c) $I_x$ için düşey şerit seçmek




$$I_x = \int dI_x \; ; \quad dI_x = dI_x' + dA.\tilde{y}^2$$

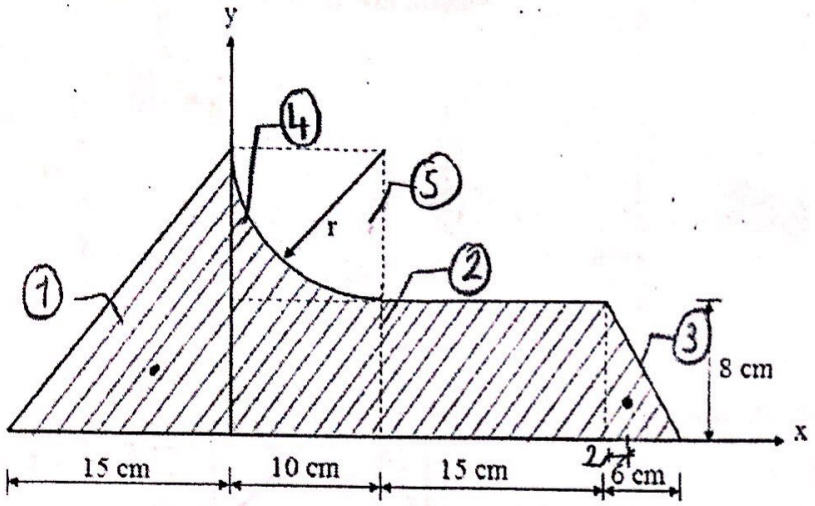
$$dI_x = \frac{1}{12}.dx.y^3 + y.dx.\left(\frac{y}{2}\right)^2 \Rightarrow dI_x = \frac{1}{3}.y^3.dx$$

$$I_x = \int_0^1 \frac{1}{3}.y^3.dx = \int_0^1 \frac{1}{3}.(x^{2/3})^3$$

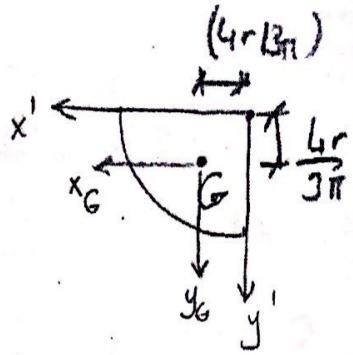
$$I_x = \int_0^1 \frac{1}{3} x^2 . dx = \frac{1}{9} = 0,111\, m^4 \;//$$

**Puan** **Soru**

**12** Şekil 2'de verilen taralı alanın y eksenine göre atalet momentini hesaplayınız.




**Şekil-2**




$I_{y'} = \frac{\pi r^4}{16}$

$I_{yG} = \frac{\pi r^4}{16} - \frac{\pi r^2}{4} \cdot \left(\frac{4r}{3\pi}\right)^2$

$I_{yG} = \frac{\pi r^4}{16} - \frac{16 r^4}{36\pi}$

$I_{yG} = \left(\frac{\pi r^4}{16} - \frac{4r^4}{9\pi}\right)$

$I_{y1} = \frac{18 \cdot 15^3}{36} + \frac{15 \cdot 18}{2} \cdot 5^2 = 5062{,}5\ cm^4$

$I_{y2} = \frac{8 \cdot 25^3}{12} + 8 \cdot 25 \cdot 12{,}5^2 = 41667\ cm^4$

$I_{y3} = \frac{8 \cdot 6^3}{36} + \frac{8 \cdot 6}{2} \cdot 27^2 = 17544\ cm^4$

$I_{y4} = \frac{10 \cdot 10^3}{12} + 10 \cdot 10 \cdot 5^2 = 3333{,}33\ cm^4$

$I_{y5} = \frac{\pi \cdot 10^4}{16} - \frac{\pi \cdot 10^2}{4} \cdot \left(\frac{4 \cdot 10}{3\pi}\right)^2 + \frac{\pi \cdot 10^2}{4} \cdot \left(10 - \frac{4 \cdot 10}{3\pi}\right)^2$

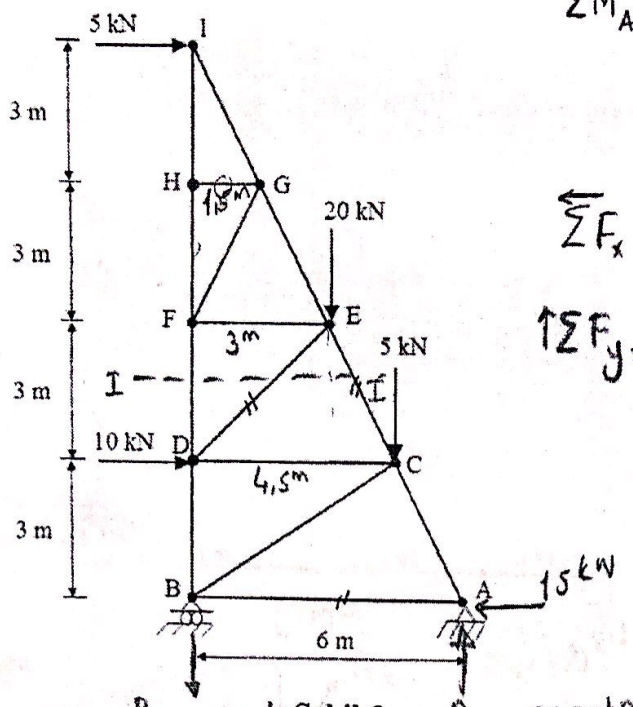
$I_{y5} = 3150{,}8\ cm^4$

$I_y = 5062{,}5 + 41667 + 17544 + 3333{,}33 - 3150{,}8$

$I_y = 64456\ cm^4$

**Puan** **Soru**

**13** Şekil 3'de verilen kafes sistemde DE, AB ve EC çubuk kuvvetlerini hesaplayınız. Çubuk kuvvetlerinin çekme ya da basınç olup olmadığını belirleyiniz.




Şekil-3

$\curvearrowleft \Sigma M_A = 0 \Rightarrow R_{By} \cdot 6 + 10 \cdot 3 + 5 \cdot 12 - 20 \cdot 3 - 5 \cdot 1,5 = 0$

$R_{By} = -3,75\ kN$

$R_{By} = 3,75\ (\downarrow)$

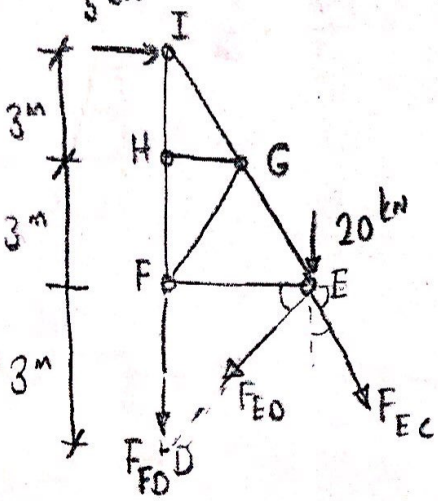
$\leftarrow \Sigma F_x = 0 \Rightarrow R_{Ax} - 5 - 10 = 0 \Rightarrow R_{Ax} = 15\ kN$

$\uparrow \Sigma F_y = 0 \Rightarrow R_{Ay} - 20 - 5 - 3,75 = 0$

$R_{Ay} = 28,75\ kN$

$R_{By} = 3,75\ kN$ $\quad R_{Ay} = 28,75\ kN$

I-I Kesimi




$\curvearrowleft \Sigma M_O = 0$

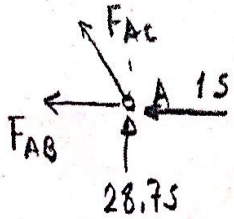
$F_{EC} \cdot \underbrace{\frac{3}{\sqrt{3^2+1,5^2}}}_{F_{ECy}} \cdot 3 + F_{EC} \cdot \underbrace{\frac{1,5}{\sqrt{3^2+1,5^2}}}_{F_{ECx}} \cdot 3 + 20 \cdot 3 + 5 \cdot 9 = 0$

$F_{EC} = -26,09\ kN$ (Basınç)

$\leftarrow \Sigma F_x = 0 \Rightarrow \underbrace{F_{EO} \cdot \frac{3}{\sqrt{3^2+3^2}}}_{F_{EO}} - 5 + 26,09 \cdot \frac{1,5}{\sqrt{1,5^2+3^2}} = 0$

$F_{EO} = -9,43\ kN$ (Basınç)

A düğüm noktasında:



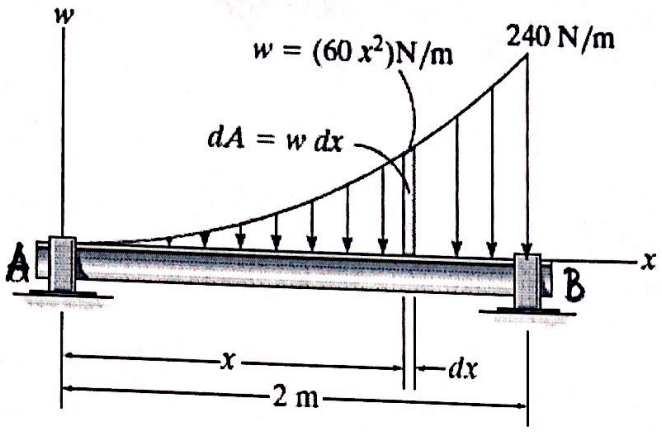

$\uparrow \Sigma F_y = 0 \Rightarrow F_{AC} \cdot \frac{3}{\sqrt{3^2+1,5^2}} + 28,75 = 0$

$F_{AC} = -32,143\ kN$

$\leftarrow \Sigma F_x = 0 \Rightarrow F_{AB} + 15 - 32,143 \cdot \frac{1,5}{\sqrt{3^2+1,5^2}} = 0$

$F_{AB} = -0,625\ kN$ (Basınç)

(14) Şekilde yükleme durumu verilen kirişin mesnet reaksiyonlarını hesaplayınız.




(a)

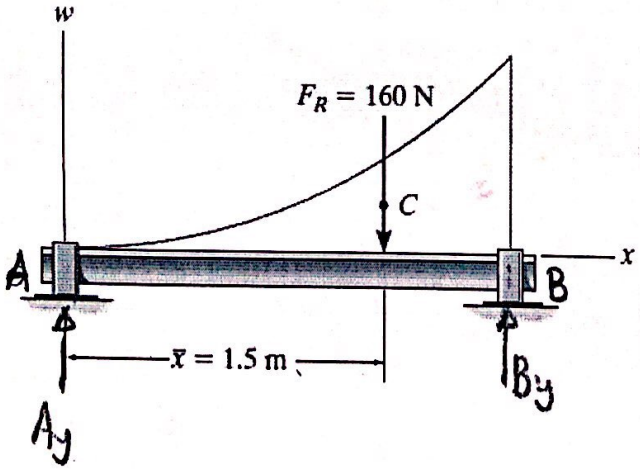
$+\downarrow F_R = \Sigma F;$

$$F_R = \int_A dA = \int_0^{2\,m} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Big|_0^{2\,m} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$F_R = 160\ N$

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\,m} x(60x^2)\,dx}{160\ N} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\,m}}{160\ N} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160\ N}$$

$\bar{x} = 1.5\ m$

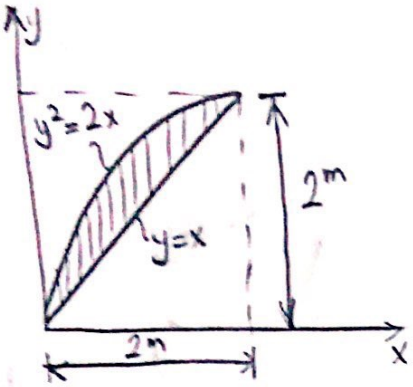



$+\circlearrowleft \Sigma M_A = 0 \Rightarrow B_y \cdot 2^m - 160 \cdot 1,5 = 0 \Rightarrow B_y = 120^N\ (\uparrow)$

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y + B_y - F_R = 0 \Rightarrow A_y + 120 - 160 = 0$

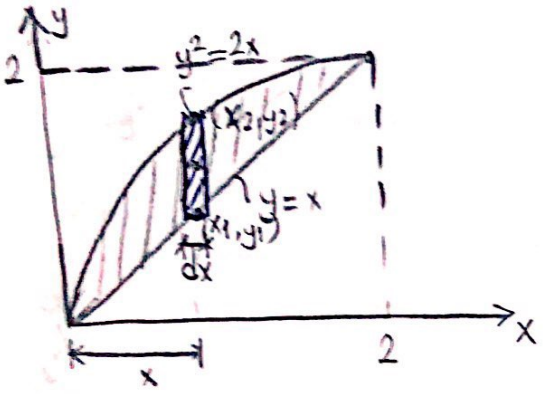
$A_y = 40^N\ (\uparrow)$

11)




Şekilde verilen taralı alanın y eksenine göre atalet momentini ve ağırlık merkezinin y koordinatını hesaplayınız.

$I_y$ için
Düşey şerit seçilirse




$$I_y = \int x^2 . dA \; ; \quad dA = (y_2 - y_1).dx$$

$$y_2 = (2x)^{1/2} \; , \; y_1 = x$$

$$dA = [(2x)^{1/2} - x].dx$$

$$I_y = \int_0^2 x^2 . [(2x)^{1/2} - x].dx$$

$$= \int_0^2 (\sqrt{2} . x^{5/2} - x^3) dx$$

$$= \left( \frac{2}{7} . \sqrt{2} . x^{7/2} - \frac{x^4}{4} \right) \Big|_0^2$$

$$I_y = 0{,}571 \, m^4$$

$$\bar{y} = \frac{\int y_{el} dA}{\int dA} \; ; \quad dA = (y_2 - y_1).dx \; ; \quad y_2 = (2x)^{1/2} \; , \; y_1 = x$$

$$dA = [(2x)^{1/2} - x].dx$$

$$y_{el} = \frac{(y_2 - y_1)}{2} + y_1 = \left( \frac{y_2 + y_1}{2} \right)$$

$$y_{el} = \frac{(2x)^{1/2} + x}{2}$$

$$\int_0^2 y_{el} . dA = \int_0^2 \frac{[(2x)^{1/2} + x]}{2} . [(2x)^{1/2} - x)] dx$$

$$= \int_0^2 \frac{(2x - x^2)}{2} . dx$$

$$\int_0^2 x . dA = \left( \frac{x^2}{2} - \frac{x^3}{6} \right) \Big|_0^2 = 0{,}667 \, m^3$$

$$dA = \int_0^2 [(2x)^{1/2} - x].dx$$

$$dA = \left( \sqrt{2} . \frac{2}{3} . x^{3/2} - \frac{x^2}{2} \right) \Big|_0^2$$

$$dA = 0{,}667 \, m^2$$

$$\bar{y} = \frac{\int_0^2 y_{el} . dA}{\int_0^2 dA} = \frac{0{,}667}{0{,}667} = 1$$